

**Exercise 23**

Regard  $y$  as the independent variable and  $x$  as the dependent variable and use implicit differentiation to find  $dx/dy$ .

$$x^4y^2 - x^3y + 2xy^3 = 0$$

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**Solution**

Differentiate both sides of the given equation with respect to  $y$ .

$$\frac{d}{dy}(x^4y^2 - x^3y + 2xy^3) = \frac{d}{dy}(0)$$

$$\frac{d}{dy}(x^4y^2) - \frac{d}{dy}(x^3y) + \frac{d}{dy}(2xy^3) = 0$$

$$\left[ \frac{d}{dy}(x^4) \right] y^2 + x^4 \left[ \frac{d}{dy}(y^2) \right] - \left[ \frac{d}{dy}(x^3) \right] y - x^3 \left[ \frac{d}{dy}(y) \right] + \left[ \frac{d}{dy}(2x) \right] y^3 + 2x \left[ \frac{d}{dy}(y^3) \right] = 0$$

$$\left[ (4x^3) \cdot \frac{d}{dy}(x) \right] y^2 + x^4(2y) - \left[ (3x^2) \cdot \frac{d}{dy}(x) \right] y - x^3(1) + (2x')y^3 + 2x(3y^2) = 0$$

$$4x^3y^2x' + 2x^4y - 3x^2yx' - x^3 + 2y^3x' + 6xy^2 = 0$$

Solve for  $x'$ .

$$(4x^3y^2 - 3x^2y + 2y^3)x' = -2x^4y + x^3 - 6xy^2$$

$$x' = \frac{-2x^4y + x^3 - 6xy^2}{4x^3y^2 - 3x^2y + 2y^3}$$